Finite Elements for Microwave Device Simulation: Application to Microwave Dielectric Resonator Filters

Jean-Pierre Cousty, Serge Verdeyme, Michel Aubourg, and Pierre Guillon, *Associate Member, IEEE*

***Abstract*—The two dimensional (2-D) and three dimensional (3-D) finite element method (FEM) are applied to compute the exact scattering parameters (taking into account the transmission lines) of dielectric resonator (DR's) filters. These filters are acting both on TM or hybrid modes. A sensitivity analysis is also performed to predict the influence of the geometrical parameters on the devices responses. Examples of one, two and three DR filters are given. Experimental results are in good agreement with theoretical ones.**

I. Introduction

Dielectric resonators (DRs) made from highly temperature stable low-loss ceramics are finding increasing microwave applications due to their desirable properties and their commercial availability at reasonable prices. Miniaturization of components is a major driving factor in the use of these ceramics. Several applications require the knowledge of their electromagnetic parameters with a high degree of accuracy. Analysis methods, whose merits are described and compared in [3], have recently received considerable attention. For example, there are the perfect magnetic conducting wall methods [1] and the dielectric waveguide method [2] as well as their perturbational correction and variational improvements [3], various radial and axial mode matching methods [4]–[6], the asymptotic expansion method [7]–[8], for resonators with very high permittivities, the moment method based on the surface integral techniques [9]–[10], and the general mode matching approaches using dyadic Green functions or transverse modes in expending the interior and exterior fields [11]–[12].

These methods are applied to compute rigorously the resonant frequencies and the electromagnetic field distribution of axis symmetric structures, and then to deduce their unloaded quality factors. To evaluate the generally required S-parameters, numerous simplifications have been developed. The DR coupling with a transmission line [13]–[14] or the coupling between DR's [15] are characterized efficiently for some particular geometries. However, for several practical microwave circuits, these assumptions do not hold and a 3 dimensional (3-D) analysis is required to investigate the overall structure, and the effects on the filters parameters of the transmissions lines, the metallic enclosure, the inhomogeneous dielectric distribution, etc.

A. Christ *et al.* [16] have implemented a 3-D finite difference CAD package to compute the coupling of a DR with a microstrip line. In the present paper, we use the 2-D and the 3-D finite element method (FEM) for the investigation of both TM and hybrid DR modes. This method, applied to solve the free oscillation wave equation, has already been presented in [17] and [18]. It permits an accurate evaluation of DR resonant frequencies, field distributions, unloaded quality factors and couplings between DR's. It has been adapted to solve the forced oscillation problem [19] and is used to evaluate the precise loaded quality factor, the spurious modes position; it is now applied to design high quality DR filters.

Section II describes succinctly the method and its mathematic formulation. It is applicable to a rigorous calculation of general microwave circuit discontinuities. The example of a T junction is performed to validate the S-parameters obtained.

In Section III, the method is applied to DR devices characterization. The 2-D FEM is used to determine the S-parameters of a one, a two and a three poles DR filters coupled by coaxial probes on their $TM_{01\delta}$ mode. A sensitivity analysis is developed to show the geometrical parameters influence on these devices and is used to tune the three-pole DR filter.

A DR acting on its first hybrid modes is also analyzed by using the 3-D FEM. These theoretical results are compared with experimental ones.

II. The Finite Element Method

The geometry of the closed inhomogeneous structure (volume V) under test is given in Fig. 1. The dielectric contained in it is assumed to be linear, lossless, isotropic, and of arbitrary shape. This structure can be excited by microstrip lines, coaxial lines or metallic guides. Those transmission lines are limited by p planes P_k (surface S_k) called reference planes, where the scattering parameters are calculated. The half infinities of lines removed are

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The authors are with the Universite de Limoges, Institut de Recherche en Communications, 123 Avenue Albert Thomas, 87060 Limoges Cedex, France.
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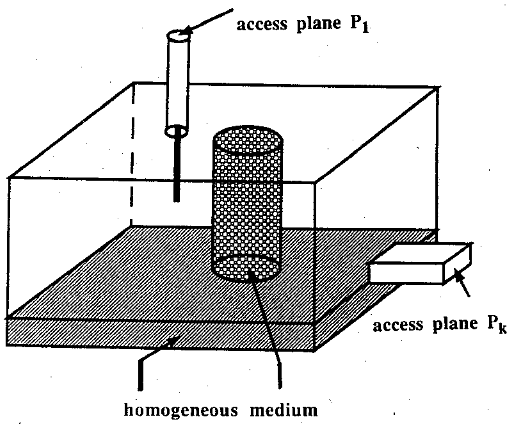


Fig. 1. A loaded 3-D structure.

substituted by electric $\overrightarrow{J_{eP_k}}$ and magnetic $\overrightarrow{J_{mP_k}}$ surface distribution currents. These currents can be developed to introduce voltage current wave a_k and b_k.

This structure is characterized using the finite element method. A detailed formulation is presented in [20]–[22]. This technique is applied, assuming the time dependence of the form $e^{j\omega t}$, to solve the following vectorial wave equation, derived from Maxwell's equations:

$$\iiint_V \left(\frac{1}{P}\{\vec{\nabla}\}\vec{\psi}\right)(\{\vec{\nabla}\}\vec{\phi})\, dV - k_0^2 \iiint_V q\vec{\psi}\cdot\vec{\phi}\, dV = -j\omega u \sum_{k=1}^{P} \iint_{S_k} \overrightarrow{J_{S_k}}\cdot\vec{\phi}\, dS_k \tag{1}$$

with

$$k_0^2 = \omega^2\epsilon_0\mu_0$$

$\{\vec{\nabla}\}$ the rotational operator applied to a function.

For a magnetic field H formulation:

$\vec{\psi} = \vec{H}$,

$\vec{\phi} = \overrightarrow{\phi_m}$ test function normal to the magnetic surfaces

$p = \epsilon_r,\quad q = \mu_r,\quad u = \epsilon_0,\quad \overrightarrow{J_{S_k}} = \overrightarrow{J_{mS_k}}$.

For an electric field E formulation:

$\vec{\psi} = \vec{E}$,

$\vec{\phi} = \overrightarrow{\phi_e}$ test function normal to the electric surfaces

$p = \mu_r,\quad q = \epsilon_r,\quad u = \mu_0,\quad \overrightarrow{J_{S_k}} = \overrightarrow{J_{eS_k}}$.

Dividing the structure into triangular (2-D) or tetrahedron (3-D) subdomains, the application of the standard FEM technique [23] gives the following global matrix equations, for an own propagating mode in each access plane k:

$$([A] - k_0^2[B])\{H\} = \sum_{k=1}^{P} \{C\}(a_k + b_k) \tag{2}$$

$$([A'] - k_0^2[B'])\{E\} = \sum_{k=1}^{P} \{C'\}(a_k - b_k) \tag{3}$$

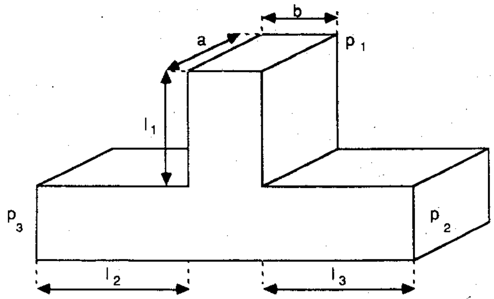


Fig. 2. T junction. a = 7.112 mm, b = 3.556 mm, l_1 = 6 mm, $l_2 = l_3$ = 2.222 mm.

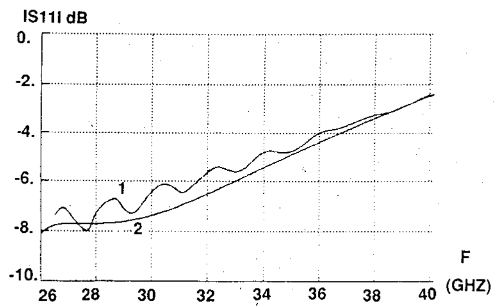


Fig. 3. $|S_{11}|$ parameters of the T junction as a function of the frequency. 1—experimental curve. 2—FEM curve.

where matrix $[A]$, $[A']$, $[B]$, $[B']$ and vectors $\{C\}$, $\{C'\}$ are tied to the first, second and third terms of (1). $\{H\}$ and $\{E\}$ are forced to satisfy boundary conditions at the domain limits.

The solutions of these equations may include many spurious modes, when classical Lagrange polynomials are used to approximate $\{E\}$, $\{H\}$ and the test functions. Those unknown functions are now approximated by first or second order mixed elements [24], which eliminate the spurious responses, and also reduce computing time [17].

The following theoretical results have been performed on a HP 9000/835.

A. Forced Oscillation Analysis

Knowing the frequency, the computation of the forced oscillation systems (2) and (3) yield to:

the scattering parameters in the access planes P_k,
the electromagnetic field distribution in the structure.

We have considered the example of a waveguide T junction (Fig. 2) in order to confirm the validity of this presented FEM formulation. Experimental results (presented by O. Picon [25]) and theoretical ones are in good agreement (Fig. 3).

Our code may be used to analyze 3-D transmission lines discontinuities.

B. Free Oscillation Analysis

A particular case of the FEM analysis is the free oscillation one. The excitation is taken out of consideration and so the reference planes P_k are short circuited. The

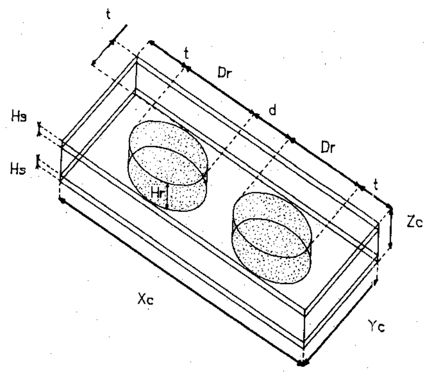
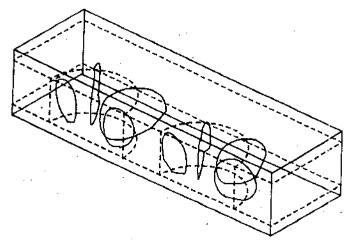
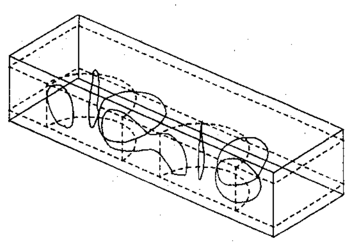


Fig. 4. Coupling between two shielded DR: $R_r = 6$ mm, $H_s = 1.5$ mm, $X_c = 42$ mm, $d = 6$ mm, $H_r = 6$ mm, $\epsilon_s = 2.2$, $Y_c = 24$ mm, $t = 6$ mm, $\epsilon_r = 36$, $Z_c = 9$ mm, $\epsilon_0 = 1$.



(a)



(b)

Fig. 5. Magnetic field lines (d = 6 mm). (a) Of the even mode: F_{oe} = 5.184 GHz. (b) Of the odd mode: F_{om} = 5.157 GHz.

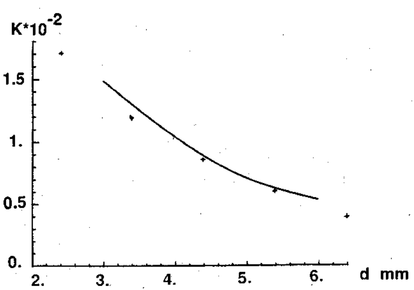


Fig. 6. Coupling coefficient k between two DR: + experimental result, —— 3-D FEM results.

second member of (1) cancels and (2) and (3) becomes

$$([A] - k_0^2[B])\{H\} = 0 \tag{4}$$

$$([A'] - k_0^2[B'])\{E\} = 0 \tag{5}$$

k_0^2 is the eigenvalue of this equation. The free oscillation analysis yields then to each eigen mode resonant frequency and field repartition of 3-D structures. Results have already been presented in [17] and [18]. An example is given here where two adjacent DR's are coupled on their $TE_{01\delta}$ mode (Fig. 4). The coupling coefficient is evaluated from [28]:

$$k_0 = \frac{f_{oe}^2 - f_{om}^2}{f_{oe}^2 + f_{om}^2} \tag{6}$$

where f_{oe} and f_{om} are respectively the frequencies of the even and odd modes of the coupled DR's structure. The magnetic field lines of these modes are shown on Fig. 5. The three dimensional FEM analysis results are compared to experimental ones on Fig. 6.

III. Analysis of Dielectric Resonators Filters

The 3-D FEM is then used for a rigorous evaluation of electromagnetic and electric parameters of three dimensional structures, but it requires usually large capacity computers. So, it will be important to take into account symmetries to reduce the analysis domain to a part of the whole structure. In particular, the evaluation of S-parameters of uniform transmission lines or of axis symmetric structures may only require the analysis of a generating section by the 2-D FEM [19] [26].

This section presents the rigorous study of different filters composed of 1, 2 or 3 dielectric resonators (DR). The first part is devoted to axis symmetric structures excited on the $TM_{01\delta}$ mode [30]. In a second part, the first hybrid modes are characterized by the 3-D FEM.

A. Axis Symmetric Structures

1. One DR Structure: The first structure under consideration is shown in Fig. 7. A cylindrical DR of radius r = 11 mm, height L = 7 mm and permittivity ϵ_r = 36.8 is enclosed in a perfectly conducting cylindric cavity of radius $r' = 2r$ and height $L' = 3L$. This DR is supported by a concentric low relative dielectric constant material ring (ϵ_s = 2, 2).

The DR is excited on its electrical dipolar $TM_{01\delta}$ mode by mean of coaxial probes mounted axially in the center of the metallic cavity. The reference planes P_1 and P_2 are chosen far enough from the discontinuities, so that the higher TM modes of the probes vanish in those planes where the scattering matrix parameters are computed.

The theoretical S_{21} modulus variation as a function of the frequency for a 6 mm probe depth penetration is presented in Fig. 8. Since the "theoretical structures" are supposed lossless, the theoretical external quality factor Q_e of the structure can be obtained from the pass band response of the one pole DR filter, applying [27]:

$$Q_e = \frac{\omega_0}{\Delta\omega} \tag{7}$$

where:

ω_0 is the resonant frequency
$\Delta\omega$ is the pass band width.

To evaluate the loaded quality factor Q_L, we first evaluate the unloaded quality factor Q_0 by solving the free

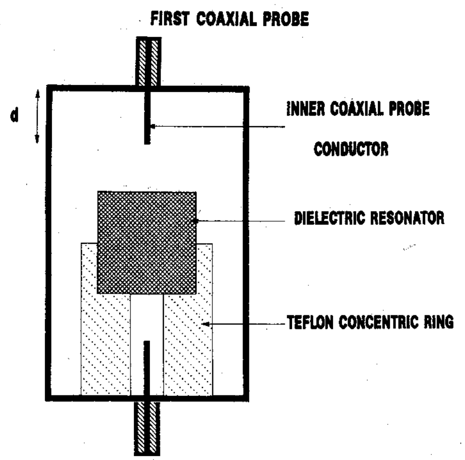


Fig. 7. A dielectric resonator coupled with two coaxial probes. DR: r = 11 mm, L = 7 mm, ϵ_r = 36.8. Cavity: r' = 22 mm, L' = 21 mm.

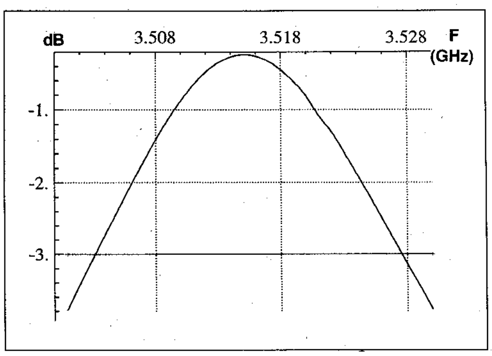


Fig. 8. $|S_{21}|$ parameter as a function of the frequency for a 6 mm probe depth penetration. f_0 = 3.5155 GHz, Δf = 23 MHz, Q_e = 153.

oscillation equation [18], and knowing Q_e from (7), we deduce Q_L from (8).

$$Q_L^{-1} = Q_0^{-1} + Q_e^{-1}. \tag{8}$$

A comparison of theoretical and experimental Q_L factor for different coaxial probe depth penetrations is presented in Fig. 9. Fig. 10 shows the $|S_{21}|$ parameter variations for some probe penetrations. The forced oscillations resonant frequencies have been compared to free oscillations ones. We can deduce from these results that the input and output couplings don't disturb the devices resonant frequencies. These frequencies are only affected by the presence of the probes metallic perturbations which are taken into account in the free oscillation computation.

Filter designers may study the influences of the structures dimensions on the filter responses to know the degree of accuracy required on them. These influences can be predicted by a theoretical sensitivity calculation, which has already been presented in [19].

Considering the structure presented in Fig. 7, it yields to the scattering matrix terms derivate as a function of:

geometrical parameters:
- cavity height and radius
- probe depth penetration
- resonators height and radius

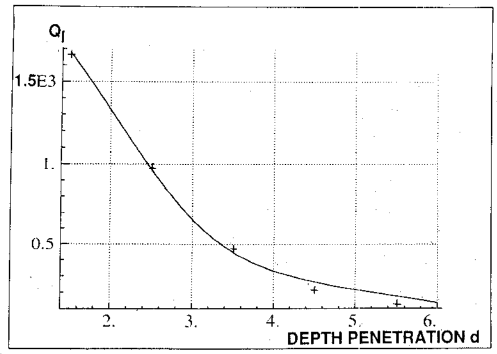


Fig. 9. Loaded quality factor of the one DR structure for different probes penetration. ——— calculated, + measured.

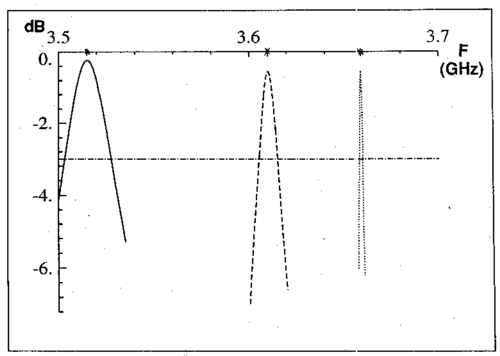


Fig. 10. S_{21} modulus parameter as a function of the frequency for various probe depth penetration. ——— d_p = 6 mm, --- d_p = 4 mm, ········ d_p = 2 mm, * free oscillation resonant frequencies.

electrical parameters:
- DR permittivity
- support permittivity

Note that we can also compute sensitivity relative to many parameters at the same time. The computer algorithm will provide in a few seconds accurate results for structures slightly different from the original one.

To appreciate the accuracy of the method, we have drawn on Fig. 11, both an interpolated curve obtained for $\Delta\epsilon_r$ = +0.01, and the computed response for $\epsilon_r = \epsilon_{r1} + \Delta\epsilon_r$, calculated by using a new mesh of the structure and running again the program.

2. *Two DR's Structure:* Two dielectric resonators are included in a perfectly conducting cylindric shield. They are supported by a concentric Teflon ring. The excitation is achieved by coaxial probes mounted axially in the center of the cavity (Fig. 12).

Fig. 13 presents some responses obtained for various distances d between DR's as a function of the frequency. We can verify that the coupling coefficient [28] decreases when d increases. This coefficient, solution of the forced oscillation problem, is compared to the free oscillation one [17] (Fig. 14).

3. *Three DR's Structures:* In designing a band pass filter, we first determine the set of coupling coefficients required to obtain the desired band width and response shape [29]. We project here to realize a 3 order Butterworth function filter (f_0 = 4113 MHz, Δf = 80 MHz). Its synthesis gives the parameters listed in Table I.

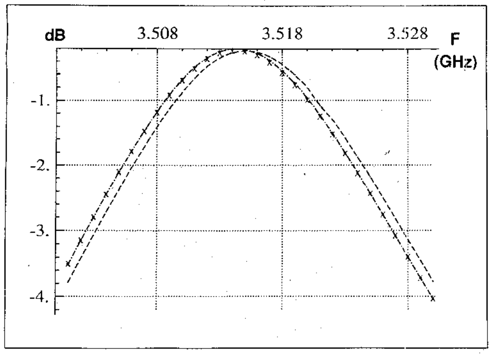


Fig. 11. $|S_{21}|$ parameter as a function of the frequency. --- computed with $\epsilon_r = 36.8$, *** interpolated with $\Delta\epsilon_r = +0.01$, — — computed with $\epsilon_r = 36.81$.

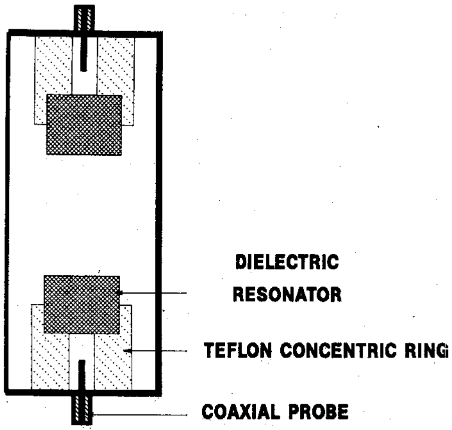


Fig. 12. Two dielectric resonators coupled with two coaxial probes. DR's: $r = 11$ mm, $L = 7$ mm, $\epsilon_r = 36.8$. Cavity: $r' = 22$ mm, $L' = (4 \times 7 + d)$ mm.

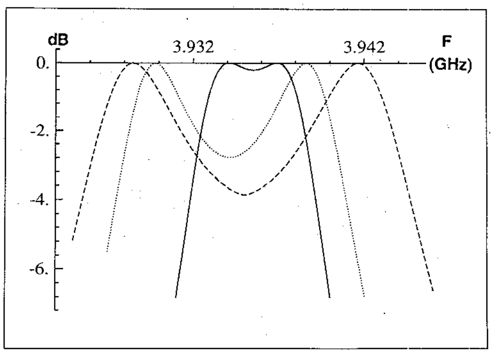


Fig. 13. $|S_{21}|$ parameter as a function of the frequency for various distances d between two DR's. —— $d = 80$ mm, — — $d = 70$ mm, ······ $d = 64$ mm.

The structure chosen to realize this function is shown on Fig. 15. Each ending DR is coupled to a coaxial probe. Cascaded DR's are coupled between them by means of metallic circular irises. Up to now, the structure dimensions required to respect the synthesis results are determined as follows.

The dimensions of the DR and the cavity radius are computed to determine the central filter frequency. An own DR is then enclosed in a metallic box as shown in Fig. 7. To obtain $f_0 = 4200$ MHz, the free or forced os-

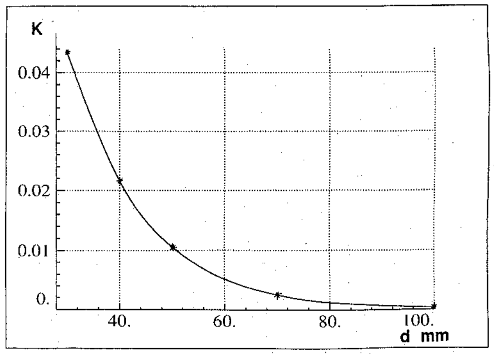


Fig. 14. The coupling coefficient between two DR's as a function of the distance d. --- free oscillation results, *** forced oscillation results.

TABLE I
COUPLING COEFFICIENTS BETWEEN DR'S

	1	2	3
1	0	$1.077 \cdot 10^{-2}$	0
2	$1.077 \cdot 10^{-2}$	0	$1.077 \cdot 10^{-2}$
3	0	$1.077 \cdot 10^{-2}$	0

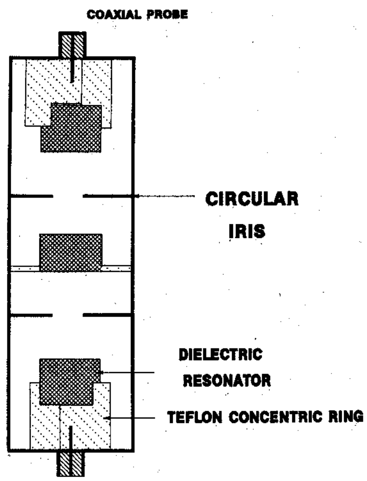


Fig. 15. Three DR's coupled with two coaxial probes. DR's: $r = 11$ mm, $L = 7$ mm, $\epsilon_r = 36.8$. Iris: opening radius: 7 mm, thickness: 1 mm. Cavity: $r' = 22$ mm, $L' = 21$ mm.

cillations 2-D FEM gives:

$$\text{DR: } H_r = 7 \text{ mm} \qquad \text{Cavity: } D_c = 44 \text{ mm}$$

$$D_r = 22 \text{ mm}$$

The probes dimensions are also computed, to respect the input output coupling coefficients defined previously (Table I). This analysis yields to the probe depth penetration $d_p = 3.5$ mm.

The DR's spacing and irises dimensions are computed by the free oscillation FEM on structures composed of only two DR's [17]. This method gives:

$$\text{Distance between DR's 1 and 2: } d_{12} = 7 \text{ mm}$$

$$\text{DR's 2 and 3: } d_{23} = 7 \text{ mm}$$

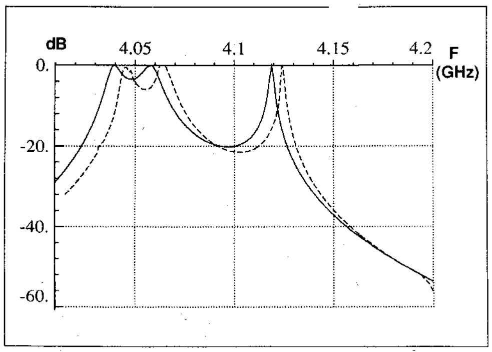


Fig. 16. $|S_{21}|$ parameter of the three DR filter. —— first analysis results, ---- sensitivity computations.

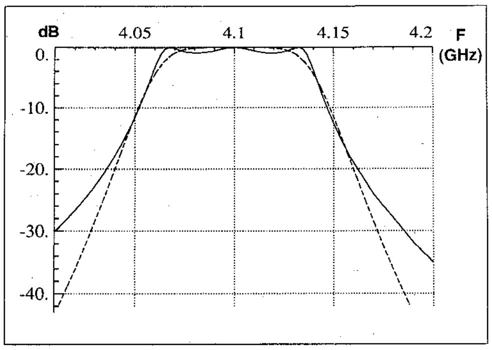


Fig. 17. $|S_{21}|$ parameter of the three DR filter. —— correct response, --- butterworth theoretical curve. Mesh: 352 elements, 3-order polynomial. Computing time: 27 s per frequency.

Dimensions of:
- upper iris:
  - opening radius: 7 mm
  - thickness: 1 mm
- lower iris:
  - opening radius: 7 mm
  - thickness: 1 mm

When such a structure is built and measured, tuning adjustments have to be made by means of metallic screws approaching the DR's and the irises, to respect the theoretical requirements. Some important parameters, like DR's influences on each other, are not taken into account in such theoretical analysis.

We propose now to deal with the whole structure. For that, we use the 2-D forced oscillation FEM and we obtain the non adequate response shown on Fig. 16 (the dimensions are the same that those used previously). We have then tried to tune our filter using the sensitivity analysis, which shows that an adjustment of the DR's positions quickly yields to the required filter curve shown on the Fig. 17.

This example confirms that the study of each parameter of the structure separately does not provide the exact dimensions required to build the synthesized filter. When a great accuracy is desired in the designing, the rigorous analysis of the whole structure is required.

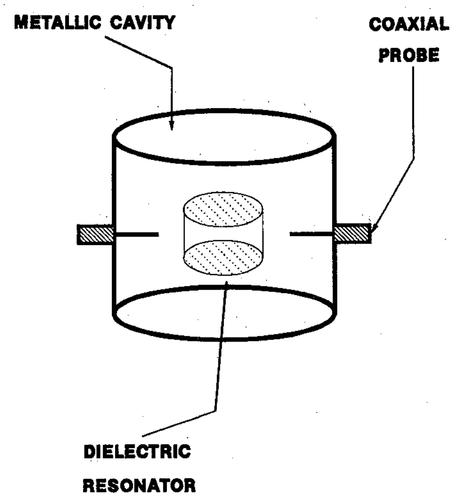


Fig. 18. DR acting on its hybrids modes coupled to two coaxial probes. DR: r = 17 mm, L = 7 mm, ϵ_r = 36.8. Cavity: r' = 22 mm, L' = 21 mm.

IV. Three Dimensional Structure

The 2-D forced oscillations FEM have been applied to characterize rigorously axis symmetric structures acting on the TM_0 mode. The transmission lines required to couple the structures on their TE or hybrid modes can't respect this axis symmetry. If those lines are considered as negligible perturbations, a 2-D free oscillations analysis is efficient to determine resonant frequency and unloaded quality factor of the structure modes. However, a 2-D forced oscillations analysis can't handle the S-parameters. To treat this problem or to characterize asymmetric devices, we use the 3-D forced oscillations method.

An example of structure we have studied is shown on Fig. 18. The DR (L = 7 mm, r = 17 mm) shielded in a circular metallic box (L' = 21 mm, r' = 22 mm) is connected to two coaxial probes placed in the lateral wall of the box to couple the well known electric field of the first hybrid $HEM_{11\delta}$ DR mode.

In Fig. 19(a) and (b), the theoretical and experimental $|S_{21}|$ parameters have been drawn as a function of the frequency. From these curves, we can evaluate the theoretical:

resonant frequency $f_0^t = 3.554$ GHz
external quality factor $Q_e = 507$

The unloaded quality factor Q_0 of the structure computed by means of the 3-D FEM gives: $Q_0 = 3700$. From (8), we compute $Q_L = 446$.

From the experimental results (Fig. 19), we obtain the resonant frequency $f_0 = 3.449$ GHz and the loaded quality factor $Q_L = 459$. This result agrees very closely with experimental one.

The scattering parameters variations as a function of the frequency of the second hybrid mode of this structure have also been measured and computed. The results obtained are given in Fig. 20(a) and (b). We can note that in this case, the probes position is not correct to couple

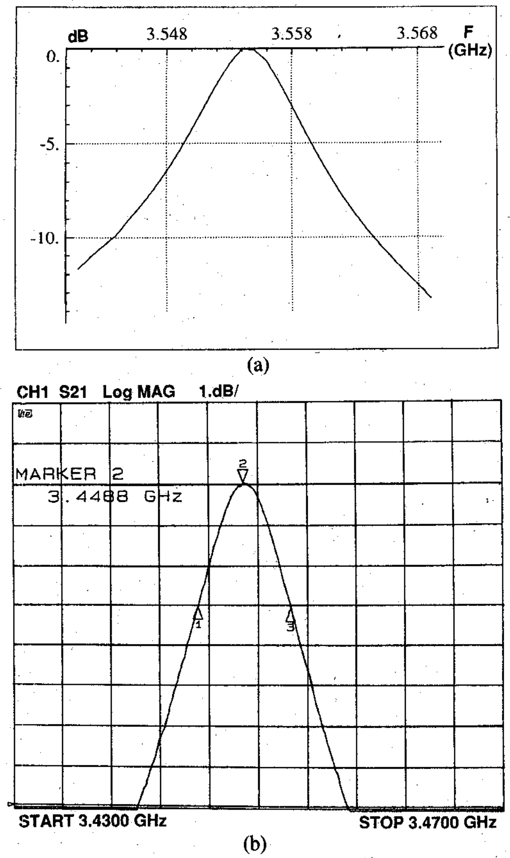


Fig. 19. $|S_{21}|$ parameter as a function of the frequency for a DR acting on its first hybrid mode. (a) Calculated. (b) Measured. Mesh: 414 tetrahedrons, 2-order mixed element. Computing time: 12.6 mn per frequency.

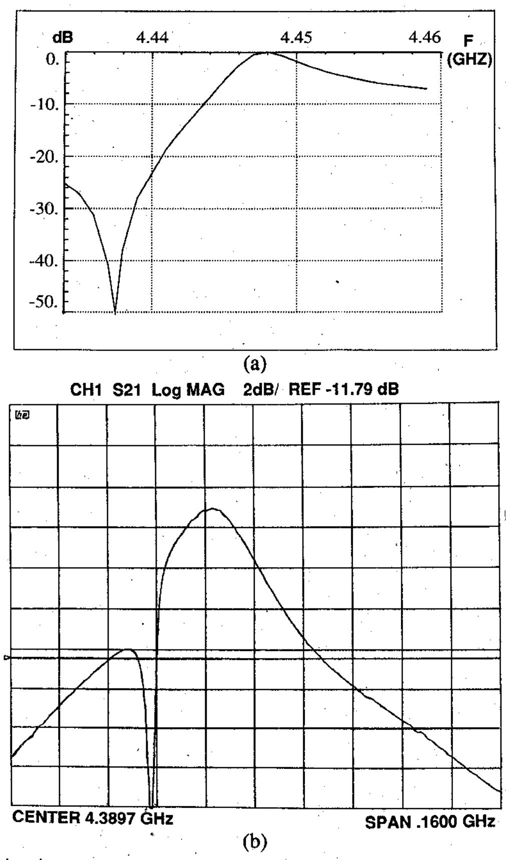


Fig. 20. $|S_{21}|$ parameter as a function of the frequency for a DR acting on its second hybrid mode. (a) Calculated. (b) Measured.

this mode, as indicated by both theoretical and experimental curves. So we can note that the theoretical FEM response permits to predict experimental ones with a good accuracy.

V. Conclusion

The FEM have been developed to characterize DR filters parameters. Good agreement between theoretical and experimental results have been observed for DR's acting on TM_0 but also on hybrid modes. The great DR dependence on its environment requires the application of this rigorous method to the whole structure analysis as it has been shown on the 3 DR's filter. Sensitivity computations may be used to correct device responses before experiment structures, then reducing deviation from theoretical results, small tuning adjustments may be required on the practical devices. The FEM is now applied to compute coupling between DR's and coaxial loops or microstrip lines, but is also applicable to analyze planar resonators and discontinuities between transmission lines.

References

[1] Okaya and Barash, "The dielectric microwave resonator," *Proc. IRE*, vol. 50, pp. 2081–2092, Oct. 1962.
[2] S. B. Cohn, "Microwave band pass filters containing high-Q dielectric resonator," *IEEE Trans. Microwave Theory Tech.*, vol. 16, pp. 218–227, Apr. 1968.
[3] D. Kajfez and P. Guillon, *Dielectric Resonators.* Norwood, MA: Artech House, 1986.
[4] D. Maystre, P. Vincent, and J. C. Mage, "Theoretical and experimental study of the resonant frequency of a cylindrical dielectric resonator," *IEEE Trans. Microwave Theory Tech.*, vol. 31, pp. 846–848, Oct. 1983.
[5] K. A. Zaki and A. E. Atia, "Modes in dielectric loaded waveguides and resonators," *IEEE Trans. Microwave Theory Tech.*, vol. 31, pp. 1039–1045, Dec. 1983.
[6] Sz. Maj and M. Pospieszalski, "A composite multilayered cylindrical dielectric resonator," in *IEEE MTT-S Int. Microwave Symp. Dig.*, San Francisco, May 1984, pp. 190–192.
[7] J. Van Bladel, "Resonances of a 'Pull-base' dielectric material," in *Proc. European Microwave Conf.*, Paris, 1978, pp. 428–432.
[8] R. De Smedt, "Correction due to a finite permittivity for a ring resonator in free space," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 1288–1293, Oct. 1984.
[9] A. W. Glisson, D. Kajfez, and J. James, "Evaluation of modes in dielectric resonators using a surface integral equation formulation," *IEEE Trans. Microwave Theory Tech.*, vol. 31, pp. 1023–1029, Dec. 1983.
[10] A. W. Glisson and D. R. Wilton, "Simple and efficient numerical methods for problems of electromagnetic radiation and scattering from surfaces," *IEEE Trans. Antennas Propagat.*, vol. 28, pp. 593–603, Sept. 1980.
[11] J. Van Bladel, "On the resonances of a dielectric resonator of very high permittivity," *IEEE Trans. Microwave Theory Tech.*, vol. 23, pp. 195–208, Feb. 1979.
[12] M. Verplanken and J. Van Bladel, "The electric dipole resonances of ring resonances of very high permittivity," *IEEE Trans. Microwave Theory Tech.*, vol. 24, pp. 108–112, Feb. 1976.
[13] A. E. Atia and R. R. Bonetti, "Coupling of cylindrical dielectric resonators to microstrip lines," in *IEEE MTT-S Int. Microwave Symp. Dig.*, Los Angeles, 1981, p. 163.
[14] P. Guillon, B. Byzery, and M. Chaubet, "Coupling parameters between a dielectric resonator and a microstrip line," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 222–226, Mar. 1985.
[15] S. B. Cohn, "Direct coupled resonator filters," *Proc. IRE*, vol. 45, pp. 187–196, Feb. 1957.
[16] A. Christ, J. M. Dieudonne, and V. Krozer, "Accurate characterization of DR coupling with new general purpose CAD program," *Electron. Lett.*, vol. 26, no. 16, pp. 1026–1028, Aug. 1989.

[17] V. Madrangeas, M. Aubourg, P. Guillon, S. Vigneron, B. Theron, and D. Parise, "A new finite element method formulation applied to DR microwave filters design," in *IEEE MTT-S Int. Microwave Symp. Dig.*, vol. 1, Dallas, 1990, pp. 411–413.
[18] S. Verdeyme and P. Guillon, "Three dimensional finite element method applied to dielectric resonator devices," in *IEEE MTT-S Int. Microwave Dig.*, June 1989, vol. 3, pp. 1151–1154.
[19] E. Marouby, M. Aubourg, and P. Guillon, "Application of finite element method to the study of transition between coaxial lines," *Proc. Inst. Elec. Eng.*, vol. 137, pt. H, no. 4, Aug. 1990.
[20] J. P. Webb, "The finite element method for finding modes of dielectric loaded cavities," *IEEE Trans. Microwave Theory Tech.*, vol. 33, pp. 635–639, July 1985.
[21] A. Konrad, "A direct three dimensional finite element method for the solution of electromagnetic fields in cavities," *IEEE Trans. Magn.*, vol. MAG-21, pp. 2276–2279, Nov. 1985.
[22] K. Ise, K. Inoue, and M. Koshira, "Three dimensional finite element solution for dielectric scattering obstacles in a rectangular waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. 38, no. 9, Sept. 1990.
[23] O. C. Zienkiewicz, *The Finite Element Method in Engineering Science*. London: McGraw-Hill, 1971.
[24] J. C. Nedelec, "A new family of mixed finite elements on R^3," *Numerishe Mathematik*, no. 50, pp. 57–87, 1986.
[25] O. Picon, "Traitements électromagnétiques 3D applicables aux composants passifs micro électroniques en ondes millimétriques," Thesis, University of Rennes 1, Juillet 1988.
[26] P. S. Kooi and Al, "Finite element analysis of the shielded cylindrical dielectric resonator," *Proc. Inst. Elec. Eng.*, vol. 132, pt. H, no. 1, pp. 7–16, Feb. 1985.
[27] G. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance Matching Networks and Coupling Structures*. Norwood, MA: Artech House, 1980.
[28] K. A. Zaki and C. Chen, "Coupling of non-axially symmetric hybrid modes in dielectric resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, no. 12, pp. 1136–1142, Dec. 1987.
[29] A. L. Zverev, *Handbook of Filter Design*. New York: Wiley, 1967.
[30] Y. Kobayashi and M. Minegishi, "A low-loss bandpass filter using electrically coupled high-Q $TM_{01\delta}$ DR," *IEEE Trans. Microwave Theory Tech.*, vol. 36, no. 12, pp. 1727–1732, Dec. 1988.

Jean-Pierre Cousty was born in Limoges, France, on July 25, 1965. He received the Maîtrise of Electronic and Communications from the University of Limoges in 1989.

He is currently working to obtain his Doctorate thesis at the Optics and Microwaves Communications Laboratory, University of Limoges. His studies involve dielectric resonators and their applications in microwave devices.

Serge Verdeyme was born in Meilhards, France, in June 1963. He received the Doctorat in electronics from the University of Limoges, France, in 1989.

Currently he is an Assistant Professor at the University of Limoges, working in the Optics and Microwaves Research Institute. His main field of interest is the use of the finite element method to compute electrical parameters of microwave devices.

Michel Aubourg was born in Neuvy-Saint-Sépulchre, France, on April 28, 1950. He received the Maitrise of Mathematics in 1975, the Doctorat de troisième cycle from the University of Limoges, France, in 1978, and the Doctorat d'Etat in 1985.

Since 1979, he has been Attaché de Recherche at the Centre National de Recherche Scientifique (C N.R.S.) working at the Microwave Laboratory of the University of Limoges, France. His main area of interest is application of the finite element method in microwave devices simulation.

Pierre Guillon (A'89) was born in May 1947. He received the Doctorat es Sciences degree from the University of Limoges, France, in 1978.

From 1971 to 1980, he was with the Microwave and Optical Communications Laboratory University of Limoges, where he studied dielectric resonators and their applications to microwave and millimeter-wave circuits. From 1981 to 1985, he was a Professor of Electrical Engineering at the University of Poitiers, France. In 1985, he joined again the University of Limoges, where he is currently a Professor and head of research group in the area of microwave and millimeter-wave devices.